Geometry (input)

$OD := 168.275 \cdot mn$ $wall := 14.2748 \cdot mm$ $ca := 0 \cdot in$ $SIFi := 1$ $SIFo := 1$ $FAC := 1$
(setup parameter)

Geometry Calcs.

$wt := wall - ca$ $ID := OD - 2 \cdot wt$ $A := \frac{\pi}{4} \cdot \left(OD^2 - ID^2\right)$ $I := \frac{\pi}{64} \cdot \left(OD^4 - ID^4\right)$ $Ro := \frac{OD}{2}$ $Z := \frac{I}{Ro}$

Loads (input - WW+HP, node 520[-530])

$P := 17.25 \cdot MPa$ $Fax := -279 \cdot N$ $Mi := -931 \cdot N \cdot m$ $Mo := -133 \cdot N \cdot m$ $T := -114 \cdot N \cdot m$

Stresses:

Set stress intensification factors

$ii := if(FAC \cdot SIFi > 1, FAC \cdot SIFi, 1)$ $ii = 1$

$io := if(FAC \cdot SIFo > 1, FAC \cdot SIFo, 1)$ $io = 1$

$Slp := \frac{P \cdot ID^2}{OD^2 - ID^2}$ $Slp = 38.3 \cdot MPa$

$\sigma hoop := \frac{P \cdot OD}{2 \cdot wt}$ $\sigma hoop = 101.67 \cdot MPa$

$\sigma axial := Slp + \frac{Fax}{A}$ $\frac{Fax}{A} = -0.04 \cdot MPa$ $\sigma axial = 38.26 \cdot MPa$

$Sb := \frac{\sqrt{(ii \cdot Mi)^2 + (io \cdot Mo)^2}}{Z}$ $Sb = 3.83 \cdot MPa$ $\tau := \frac{T}{2 \cdot Z}$ $\tau = -0.23 \cdot MPa$

$\sigma := Slp + \frac{Fax}{A} + Sb$ $\sigma = 42.09 \cdot MPa$

Assemble the stress components required to calculate the three dimensional stress intensity at four locations through the pipe wall.

Calculate the stresses along the pipe diameter which is perpendicular to the resultant bending moment.

1: outside surface, moment causes tension
2: inside surface, moment causes tension
3: inside surface, moment causes compression
4: outside surface, moment causes compression

Intermediate calc's: $Ri := \frac{ID}{2}$ $Ain := \frac{\pi}{4} \cdot ID^2$ $Axs := A$

$axial := P \cdot Ain + Fax$ $bend := \sqrt{(ii \cdot Mi)^2 + (io \cdot Mo)^2}$

longitudinal stress | hoop stress | radial stress (force this term negative) | shear stress

$$\sigma l := \begin{pmatrix} \frac{axial}{Axs} + \frac{bend \cdot Ro}{I} \\ \frac{axial}{Axs} + \frac{bend \cdot Ri}{I} \\ \frac{axial}{Axs} - \frac{bend \cdot Ri}{I} \\ \frac{axial}{Axs} - \frac{bend \cdot Ro}{I} \end{pmatrix}$$

$$\sigma h := \begin{bmatrix} P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ro^2} + 1\right) \\ P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ri^2} + 1\right) \\ P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ri^2} + 1\right) \\ P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ro^2} + 1\right) \end{bmatrix}$$

$$\sigma r := \begin{pmatrix} 0 \\ -P \\ -P \\ 0 \end{pmatrix}$$

$$\tau := \begin{pmatrix} \frac{T \cdot Ro}{2 \cdot I} \\ \frac{T \cdot Ri}{2 \cdot I} \\ \frac{T \cdot Ri}{2 \cdot I} \\ \frac{T \cdot Ro}{2 \cdot I} \end{pmatrix}$$

Sa and Sb are the principal 2D stresses in the plane normal to the radial direction

$$Sa(p) := \frac{\sigma l_p + \sigma h_p}{2} + \frac{\sqrt{\left(\sigma l_p - \sigma h_p\right)^2 + \left(2\cdot\tau_p\right)^2}}{2} \qquad Sb(p) := \frac{\sigma l_p + \sigma h_p}{2} - \frac{\sqrt{\left(\sigma l_p - \sigma h_p\right)^2 + \left(2\cdot\tau_p\right)^2}}{2} \qquad Sc(p) := \sigma r_p$$

Principal stresses at positions 1 to 4

$$Sq := \begin{pmatrix} Sa(1) & Sa(2) & Sa(3) & Sa(4) \\ Sb(1) & Sb(2) & Sb(3) & Sb(4) \\ Sc(1) & Sc(2) & Sc(3) & Sc(4) \end{pmatrix} \qquad Sq = \begin{pmatrix} 76.6 & 93.85 & 93.85 & 76.6 \\ 42.09 & 41.44 & 35.08 & 34.43 \\ 0 & -17.25 & -17.25 & 0 \end{pmatrix}\cdot MPa$$

sort stresses at each position (first in ascending order, then in reverse order)

$$Q(p) := sort\left(Sq^{\langle p\rangle}\right) \qquad S(p) := reverse(Q(p))$$

$$S(1) = \begin{pmatrix} 77 \\ 42 \\ 0 \end{pmatrix}\cdot MPa \qquad S(2) = \begin{pmatrix} 94 \\ 41 \\ -17 \end{pmatrix}\cdot MPa \qquad S(3) = \begin{pmatrix} 94 \\ 35 \\ -17 \end{pmatrix}\cdot MPa \qquad S(4) = \begin{pmatrix} 77 \\ 34 \\ 0 \end{pmatrix}\cdot MPa$$

maximum principal stress (S1) at position "p"

$S1(p) := S(p)_1$ $\quad S1(1) = 77\cdot MPa$ $\quad S1(2) = 94\cdot MPa$ $\quad S1(3) = 94\cdot MPa$ $\quad S1(4) = 77\cdot MPa$

$S2(p) := S(p)_2$ $\quad S2(1) = 42\cdot MPa$ $\quad S2(2) = 41\cdot MPa$ $\quad S2(3) = 35\cdot MPa$ $\quad S2(4) = 34\cdot MPa$

minimum principal stress (S3) at position "p"

$S3(p) := S(p)_3$ $\quad S3(1) = 0\cdot MPa$ $\quad S3(2) = -17\cdot MPa$ $\quad S3(3) = -17\cdot MPa$ $\quad S3(4) = 0\cdot MPa$

SI:: Maximum Shear Stress Intensity at position "p"

$$SI(p) := S1(p) - S3(p) \qquad \_3DMax := \begin{pmatrix} SI(1) \\ SI(2) \\ SI(3) \\ SI(4) \end{pmatrix} \qquad MaxStressIntensity := max(\_3DMax)$$

$$MaxStressIntensity = 111.1\cdot MPa$$

In non-fatigue applications, maximum stress intensity is limited by material yield stress. (Actually, maximum shear stress which is half of the stress intensity is limited by one half of yield.)

Another comparision - von Mises or octahedral shear stress (also known as equivalent stress since this stress calculation is equivalent to the energy of distortion calculation) is limited by yield stress times square root of 2 divided by 3 (.47Sy).

SOct::Octahedral Shear Stress (or Equivalent Stress)

$$SOct(p) := \frac{1}{3}\cdot\sqrt{(S1(p) - S2(p))^2 + (S2(p) - S3(p))^2 + (S3(p) - S1(p))^2}$$

$$OctMax := \begin{pmatrix} SOct(1) \\ SOct(2) \\ SOct(3) \\ SOct(4) \end{pmatrix} \quad MaxOctShear := \max(OctMax) \quad MaxOctShear = 45.38\cdot MPa$$

$$MaxOctShear = 45.38\cdot MPa$$

Following illustrates the four positions where stess is calculated: